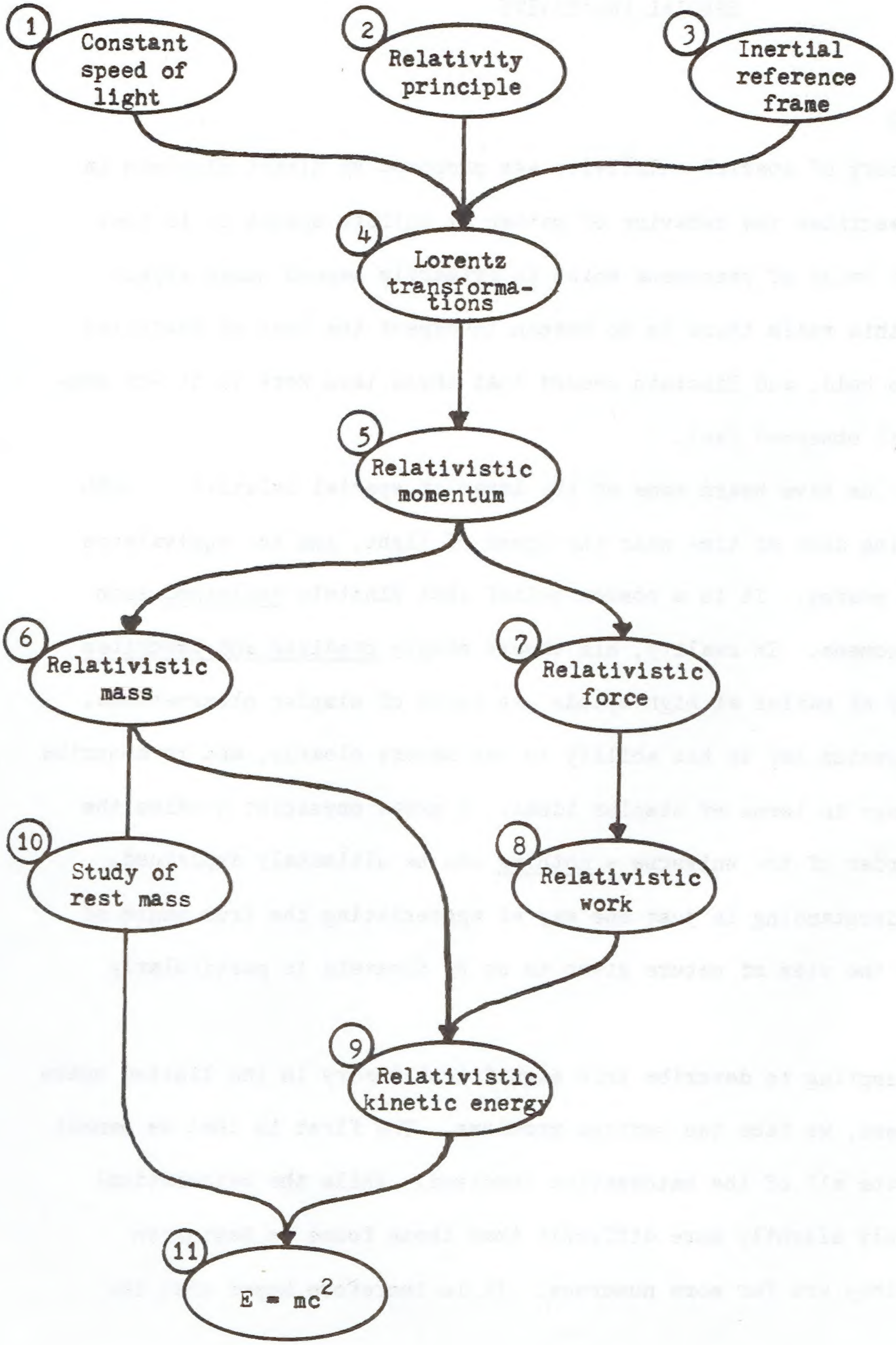
Chart #3

# SPECIAL RELATIVITY



reference frames, and the corresponding expected frequency shift. In short, the special theory of relativity is entirely self-consistent, and in particular it perfectly describes the behavior of electro-magnetic radiation.

The equation $E = mc^2$ is of course very famous, and very useful, particularly in the realm of sub-atomic particles. It is important to realize that this is not an equation of conversion of relativistic mass into energy. It is, rather, a statement of the equivalence of the two concepts. The constant $c^2$ is simply due to our choice of a measurement unit for energy, based on our original work definition. There is absolutely no way to distinguish between relativistic mass and energy, and there is really no desire to. For there is not a single bit of scientific evidence to suggest that the two concepts are at all different.

In sacrificing the apparent simplicity of Newtonian mechanics, Einstein has given us a work of pure and absolute beauty. We are now in a position to briefly review its fascinating development.

References to bibliography at end of chapter: 1-13; 4-8; 4-9; 5-3; 6-9; 6-10; 7-2.

## A Final Overview of the Logic of Special Relativity

We are now at a point where it is extremely worthwhile to review the development of Einstein's special theory of relativity. The thought behind this theory is as much a work of art as a work of science. It is hoped that the student is now in a position to perceive both these aspects of Einstein's work.

## SPECIAL RELATIVITY

Up to and including block #4 on our chart, the development of the subject is extremely deductive. By this we mean that block #4 is the only possible conclusion from the assumptions presented in blocks #1, 2 and 3. For the rest of the chart, however, this is not the case. As we have seen, this is the area which required the magnificent "guessing" of Einstein to arrive at the only reasonable conclusions. Let us now examine the relationships of our chart in detail.

Blocks #1 and #2, the constancy of the speed of light and the relativity principle, are the prime foundations of the theory of special relativity. If they were false, the rest of the theory would also be false. If, as experiment has shown, they are true, then block #4, (the Lorentz transformations), must be true. Block #3 is an essential step in our argument, not because it says anything new about the universe, but because it very carefully describes the meaning we associate to the concepts of position and time. Without this careful description, all of the discussion of special relativity would be meaningless.

As we have said, the Lorentz transformations are the inevitable conclusion from blocks #1, 2 and 3. These equations state the relationship between the measurements of two observers moving at high relative speeds. They are fairly simple, yet are very hard to believe - for they seem to contradict all common sense. The contraction of lengths and slow-down of clocks at high speeds are concepts alien to everyday experience. Nevertheless, after careful thought these results can be seen to be inevitable, and experiments in particle physics have confirmed them.

From the Lorentz transformations, we now see the development of the dynamic concepts of energy, momentum, force, acceleration, and mass.

Einstein could not deductively obtain the relationship of these concepts, however, because he had no experimental data at relativistic speeds. The development of the rest of the theory required educated guesses on the part of Einstein - guesses which were given strong support by their mutual coherentness and simplicity. Since the time of Einstein, all of these "guesses" have been verified by experiment.

The first concept developed in this way is relativistic momentum in block #5. It seemed reasonable that at all speeds there should be some vector "quantity of motion" whose sum is always conserved, and it can be easily shown that only one such vector function can be conserved in all symmetrical elastic collisions. This "relativistic momentum" is equal to the momentum of Newtonian mechanics at low speeds, but is much greater at speeds near that of light. Since he could not test his assumption, it was only a hunch on Einstein's part to correctly assume that relativistic momentum is conserved not only with symmetrical, elastic collision, but in absolutely all situations.

We next made two definitions based on the idea of relativistic momentum. The first of these is relativistic mass (block #6). This is simply a number which is a function of the Newtonian (rest) mass of an object and its speed. It is a useful number because when an object's relativistic mass is multiplied by its velocity, the result is its relativistic momentum. Thus while relativistic mass does not indicate the "quantity of matter" present, it is a very useful concept for matter in motion.

The other definition from relativistic momentum was that of relativistic force (block #7). Relativistic force is defined to be the rate of change of relativistic momentum. This definition is arbitrary, (subject only to the constraint that at low speeds it be equivalent to Newtonian force), but it is satisfying because it suggests there is a principle in relativity similar to Newton's third law of motion, (which states that action equals reaction). This is a simple and pleasing result, but does not prove anything. Nevertheless, the fact that the subsequent concepts develop so coherently is strong indication that we are following the right path. Moreover, this definition is in keeping with the behavior of high speed particles in electric fields.

We are now at the point where we make our shakiest hypothesis. In block #8, we define relativistic mechanical work to be the product of relativistic force and the distance moved in the direction of that force. Our only reason for believing this to be the case is that it is the simplest way to retain the principle of conservation of mechanical energy. Einstein had no proof he was correct here (although experiment has since verified this assumption). Once again, however, the simple coherence of the resulting three blocks were sufficient to convince Einstein that he was correct.

In block #9, we make a simple deductive conclusion from our definition of relativistic mechanical work. Simple mathematics show that the difference between the relativistic mass of an object and its rest mass is just its kinetic energy divided by the square of the speed of light. Thus we see that relativistic mass is even more useful than we previously suspected, as it increases directly with kinetic energy.

In block #10, we see that in a symmetrical, <u>inelastic</u> collision, any decrease in total kinetic energy is accompanied by an increase in the <u>rest mass</u> of the objects. In addition, by conservation of energy we feel that any decrease of kinetic energy will correspond to an equal increase of the internal energy of the bodies. Now, rest mass is our only way of defining "quantity of matter". Thus we can state that in this limited situation, an increase in the internal energy of the bodies corresponds to an increase of the rest mass. There is, in fact, no way to distinguish between these two concepts - we say they are equivalent. Einstein assumed that this result must hold in all situations, and experiments in the sub-atomic world have shown this to be correct.

In conclusion, block #11 simply ties together the equivalence of rest mass and internal energy with the correspondence of kinetic energy to the increase of relativistic mass. The result is that the total energy of a particle is given by its relativistic mass multiplied by the square of the speed of light. ($E = mc^2$.) Rest mass is thus seen to simply be energy which, at least on our level of observation, is capable of being brought to rest.

The theory we have just examined is one of the world's great works of genius. Its creation required an unbounded imagination and the noble skill of finding beautiful simplicity in the apparent chaos of nature. In this sense, Einstein was a gifted artist.